

Asset Class Portfolio Methodology

I. Asset Class Portfolio Construction: Overview

1a Modern Portfolio Theory

More than half a century has passed since Markowitz (1952, 1959) introduced the Nobel Prize-winning theory of optimal portfolio allocation in the mean-variance framework that was later dubbed Modern Portfolio Theory (MPT). Markowitz’s approach to the computation of an optimal portfolio is very elegant and, given the computational constraints at the time, analytically accessible.

Despite its academic success, PMC believes the use of MPT in the finance industry is limited. When in fact used, typically either a large set of constraints is applied or some type of methodological improvement (see below) is employed. The reason for this has to do with the extreme fluctuations of MPT optimal portfolios over time and across the risk spectrum.

To illustrate this problem, consider the following example. Suppose there are two assets in the portfolio: A and B. The correlation between the assets is assumed to be 0.9. Table 1 below displays the optimal mean-variance allocations. These results highlight the following problems inherent to MPT: (A) the optimal portfolio weight changes are extreme; and (B) the changes in the associated portfolio expected returns/standard deviations are very slight.

Table 1.

	Scenario 1			Scenario 2			Scenario 3		
	mean	std.dev.	allocation	mean	std.dev.	allocation	mean	std.dev.	allocation
Asset A	8.00%	20.00%	50.00%	8.00%	20.00%	100.00%	8.00%	20.00%	200.00%
Asset B	8.00%	20.00%	50.00%	8.01%	20.00%	0.00%	7.99%	20.00%	-100.00%
Portfolio	8.00%	19.49%	100.00%	8.00%	20.00%	100.00%	8.01%	23.66%	100.00%

These problems have also been widely documented in literature. Jobson et al (1979) and Jobson & Korkie (1980) show that mean-variance portfolios are highly sensitive to small deviations in the estimates of means and covariances (correlations and standard deviations). Best & Grauer (1991) show that optimal portfolios are very sensitive to the level of expected returns. For example, they note that “a surprisingly small increase in the mean of just one asset drives half the securities from the portfolio. Yet the portfolio’s expected return and standard deviation are virtually unchanged.”

1b Estimation Risk

There are several assumptions that are necessary for MPT to be relevant, one of which is that the means and covariances are known with certainty. Thus, if we knew the parameters with certainty and if minimizing variance while maximizing the mean of the portfolio, regardless of the resulting portfolio turnover, were the right criteria to use for our portfolio, then the above portfolio weights, however extreme, would present an appropriate strategy.

Of course, we do not know the input parameters with certainty. In fact, as noted by several authors, the estimation of these parameters is fraught with difficulty. Merton (1980) points out the expected return estimation problems by noting that a very long time series of data is required to achieve precise estimates. Similar problems can also affect the covariance estimates (Green and Hollifield, 1992; Ledoit and Wolf, 2003).

Thus, when running mean-variance optimization, the inherent instability of the suggested optimal weights is greatly exacerbated by the fact that we have to estimate the inputs, usually with a high degree of error.

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Jorion (1992) introduces the following definition for this phenomenon that he calls **estimation risk** (also known as measurement error, estimation error and parameter uncertainty): “The possibility of errors in the portfolio allocations due to imprecision on the estimated inputs to the portfolio optimization.” Michaud (1989) comments on the consequences of the optimal portfolios calculated using the traditional mean-variance optimization, calling them “estimation-error maximizers”.

1c Consequences of the Estimation Risk

What are the consequences of the estimation risk? Chopra & Ziemba (1993) investigate this question and find that errors in means are about ten times as important as errors in variances, and errors in variances are about twice as important as errors in covariances. This suggests that estimation risk can substantially impact the mean-variance optimization results.

In fact, several studies have demonstrated that the estimation risk inherent in mean-variance optimization is so large as to overwhelm any optimality of MPT optimization altogether. For example, Demiguel, Garlappi, & Uppal (2007) and Herold & Mauer (2006) show that due to the estimation error the 1/N and the market portfolio, respectively, are the best available portfolios. The basic reason for this is that the mistakes caused by using the suboptimal 1/N portfolio turn out to be smaller than the error caused by using weights from MPT (and especially the estimates of the expected rate of return) that are subject to high estimation risk.

Jagannathan & Ma (2003) argue that the minimum variance portfolio performs better than the mean-variance efficient portfolio. They note that “the sample mean is an imprecise estimator of the population mean. The estimation error in the sample mean is so large that nothing much is lost in ignoring the mean altogether when no further information about the population mean is available.”

A recent thought-provoking study from the University of Chicago (Pastor & Stambaugh, 2012) argues that even the “stocks for the long run” maxim does not hold true due to estimation risk. Previously, Campbell & Viceira (2005) had demonstrated that the “time diversification” holds when there is negative serial correlation in equity returns, but Pastor & Stambaugh (2012) show that this effect gets overwhelmed by the estimation risk inherent in the parameters.

1d Methods for Dealing with Estimation Risk

There are various approaches to dealing with the above problem. The three main methods, mentioned by Demiguel, Garlappi, & Uppal (2007) and Herold & Mauer (2006), are Bayesian estimation, bootstrap statistical approach and “heuristic” approaches.

A *Bayesian* approach allows the analyst to account for estimation risk by specifying the likely values of the parameters and the associated probabilities. This information is then used in a self-consistent portfolio optimization framework that accounts for all the available information, including estimation risk. For example, Harvey et al. (2010) propose a Bayesian framework within which to deal with the estimation risk as well as to incorporate higher moments of the return distributions.

The “*heuristic*” group of approaches contains various heuristic ways of dealing with the estimation error. An example of this is Jagannathan & Ma (2003) who investigate the effect of non-negativity constraints on the performance of the efficient portfolios.

The *bootstrap* approach accounts for estimation risk in portfolio weights by attempting to estimate this risk directly (see details of this method below). We follow the bootstrap route in constructing Envestnet’s Asset Class Portfolios (ACPs).

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II. Asset Class Portfolio Construction: Process

STEP 1: Specification of the Constraints

The first step in our estimation process is to set up portfolio constraints. While an over-constraining optimization problem (i.e., setting the constraints to obtain the desired outcome) is counterproductive, imposing a set of guiding constraints that nevertheless allow the optimization to take its course is very useful. For example, Jagannathan & Ma (2003) note the following: "It is well recognized in the literature that imposing portfolio weight constraints leads to superior out-of-sample performance of mean-variance efficient portfolios...These constraints are likely to be wrong in population and hence they introduce specification error. According to our analysis, these constraints can reduce sampling error. Therefore, gain from imposing these constraints depends on the trade-off between the reduction in sampling error and the increase in specification error."

To avoid over-constraining the optimization problem, we have imposed a number of relative constraints that are guided by the observed market capitalizations of the various asset classes. These constraints ensure that the relative sizes of the exposures in the optimized portfolio track those in the observed market portfolio, while allowing the risk/return trade-off properties of the various asset classes to shape the final portfolio. Also, we have restricted the allocation to Value and Growth to be the same, which, combined with the above size constraints, allows us to mimic the market portfolio in largely avoiding either a style- or size-bias in our Asset Class Portfolios.

Finally, we add an absolute constraint that at a 50/50 equity/fixed-income split, Intermediate Bonds should occupy no larger than 25 percent of the portfolio.

STEP 2: Bootstrap Estimation of the Mean-Variance Portfolios

It turns out that estimation risk can be expressed as equaling the bias squared of the estimated portfolio weights (when compared to the true, but unknown and unknowable portfolio weights) plus the variance of the estimated portfolio weights.¹ If we knew the true means and covariances, the Mean Squared Error (MSE) would equal zero. The goal of our estimation methodology then is to reduce the MSE as much as possible. Application of a bootstrap allows us to estimate and adjust for the bias in the mean-variance optimal weights.

The bootstrap statistical technique was first proposed by Efron (1979). Horowitz (2001) describes the bootstrap as follows: "The bootstrap is a method for estimating the distribution of an estimator or test statistic by resampling one's data or a model estimated from the data." In other words, the essence of the bootstrap is to attempt to estimate the distribution of an estimator (in our case the portfolio weights), and to do this we can employ either resampling or model-based approaches. Resampling usually turns out to be more general and expedient than the model-based approach, but we should not confuse the resampling with the general approach of the bootstrap.

Thus, the application of bootstrap statistical techniques allows us to estimate and correct for the estimation bias in the portfolio weights. We carry out this adjustment at every portfolio across the mean-variance frontier.²

Finally, we use empirical copulas (see Meucci (2011) for a brief introduction in copula statistical approach) to generate the bootstrap sample. The reason for using a copula approach is two-fold. First, it allows us to incorporate the forward-looking assumptions on means and covariances. Second, it allows us to more realistically reflect the observed idiosyncrasies (e.g., skewness, kurtosis and asymmetric tail dependence; see Patton 2004) of the return distributions.

¹ The quantity bias squared plus variance is called the Mean Squared Error (MSE). MSE is widely used in statistics as a criterion for evaluating the performance of an estimation methodology. Ordinary Least Squares (OLS) regressions are an example of being optimal in MSE sense.

² See, for example, Jorion (1992) or Lai, Xing, & Chen (2010) for the details on how to carry out this bootstrap bias adjustment.

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STEP 3: Propagation of the 50/50 Portfolio to Other Portfolio Specifications

In our asset allocation portfolio construction process, we allow for 13 domestic equity “tiers”³ or granularities at which the user can construct asset allocation portfolios. Furthermore, in our asset class portfolios we allow for the addition of up to nine diversifying asset classes⁴. Finally, there usually are no fewer than three equity/fixed-income splits (risk levels) of interest. Thus, the total number of available portfolio specifications comes out to about 19,968.⁵ This number can be slightly lower or even higher, depending on the number of equity/fixed-income splits of interest, but even under very restrictive portfolio choice assumptions, the number of available portfolio combinations that we offer is very large.

In addition, we believe the asset class portfolios should satisfy the following common sense properties across various domestic equity tiers, diversifying asset class combinations and equity/fixed-income splits:

- Domestic equity exposures should match up when moving across domestic equity tiers.
- Diversifying asset and fixed-income asset class exposures should not change when moving across domestic equity tiers.
- A particular equity/fixed-income split should be maintained when moving across domestic equity tiers and/or across diversifying asset class combinations.
- Asset class exposures should change gradually when moving across equity/fixed-income splits.

Clearly, it would be impossible to develop and maintain self-consistent optimization constraints that would deliver portfolios with these properties for the multitude of portfolio combinations that we offer. To avoid having to develop and maintain a large number of portfolio constraints and still be able to take advantage of the bootstrap bias-corrected mean-variance frontier, we proceed as follows:

- Obtain a 50/50 bootstrap-corrected portfolio from Step 2 at the most granular domestic equity level with all diversifying asset classes present.
- Designate each asset class as either equity or fixed income. This task is fairly straightforward for all asset classes present in our portfolios, except for Commodities. However, we rely on research by Idzorek (2006) to classify Commodities as an equity-like asset class, who notes that commodities behave like equities in high-inflation scenarios, but more bond-like in low-inflation scenarios.
- Propagate the 50/50 portfolio across the desired equity/fixed-income splits by maintaining the relative allocations of the equity and fixed-income portions of the 50/50 portfolio.
- If a certain diversifying asset class (either equity or fixed-income diversifying asset class) is not selected, its allocation from the 50/50 portfolio gets rolled up into the domestic equity or fixed-income allocations, respectively. The exception to this is Emerging Markets, whose allocation gets rolled into International Developed Equity, if it is present, and Domestic Equity if International Developed Equity is not present.

III. Asset Class Portfolio Construction: Results

III.A. PMC Standard Portfolios

As noted in previous sections, our Asset Class Portfolio (ACP) construction methodology allows us to construct portfolios at various domestic equity tiers and a multitude of diversifying asset class combinations. PMC Standard Portfolios represent three particular portfolios from this list and are referred to as “PMC Diversified”,

³ The available domestic equity granularity levels: All Cap, LCC+SCC, LC V/G + SCC, LCC + SC V/G, LC V/G + SC V/G, LC V/G + MC V/G + SC V/G, LCC + MCC + SCC, LC V/G + MCC + SCC, LCC + MC V/G + SCC, LCC + MCC + SC V/G, LC V/G + MC V/G + SCC, LCC + MC V/G + SC V/G, LC V/G + MCC + SC V/G. Thus, there are a total of 13 of these domestic equity levels.

⁴ Diversifying asset classes: REITs, International Developed Equity, Emerging Markets, Commodity, High Yield Bond, International Bond, Bank Loans, Emerging Markets Bonds, and TIPS, which gives us 9 diversifying asset classes.

⁵ Multiply the number of domestic equity tiers (13) by the number of possible combinations of diversifying asset classes (2 to the power of nine, which equals 6,656). Finally, multiply this product by the number of equity/fixed income splits (here 5).

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“PMC Core”, and “PMC Concentrated” ACPs. The difference between them is the size and value breakouts of the domestic equity with “PMC Diversified” having large/small cap and value/growth splits; “PMC Core” having value/growth splits only for large cap, but core allocation for the small cap; and “PMC Concentrated” having only large and small cap core exposures.

There are three main uses of PMC Standard Portfolios:

1. As reference portfolios for use in risk scoring on the Envestnet platform (see “Risk Scoring on the Envestnet Platform” white-paper for details).
2. As the basis for various managed solutions (e.g., Sigma Mutual Fund Solution, PMC Strategic ETF Solution, PMC Select Strategic Portfolios, Enterprise Solutions, etc.) and PMC’s consulting framework.
3. As starting positions for asset allocation portfolios provided via the platform to advisors, who can then further customize these according to their needs.

PMC Standard Portfolios consist of domestic equity, domestic fixed income (intermediate and short bonds), international developed equity, and international developed fixed income asset classes.

The reasoning for selecting these asset classes is two-fold. First, we want the selected asset classes to represent most of the market capitalization in the world portfolio. Domestic equity and fixed income combined with developed equity and fixed income represents 111 trillion of market capitalization (as of the beginning of 2022), which is about 22 percent of the market capitalization of the world portfolio.

Second, we want the PMC Standard Portfolios to represent long-term strategic allocations, where the choice of the diversifying asset classes is not a function of tactical or dynamic decisions (e.g., including asset classes as a result of relative underpricing or momentum considerations), but rather serves as a strategic foundation for the overall portfolio. This is especially important for the risk scoring and managed solutions uses of PMC Standard Portfolios mentioned above.

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IV. References

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V. Glossary of Terms

Bayesian statistical approach. A statistical framework that allows for consistent integration of various sources of information. A Bayesian approach allows for integration of data (e.g., returns for an asset class) with external information, such as uncertainty about the model parameters (i.e., mean, standard deviation, correlation, etc.) and other views imposed by an analyst.

Bootstrap estimation method. The bootstrap is a statistical method for estimating the distribution (i.e., histogram) of an estimator (such as sample mean or portfolio weight) by resampling the observed sample data or a model estimated from the data.

Correlation. A statistical measure of how two securities move in relation to each other. Perfect positive correlation (a coefficient of +1) implies that as one security moves, either up or down, the other security will always move in the same direction. Perfect negative correlation (a coefficient of -1) means that if one security moves up or down, the negatively correlated security will always move in the opposite direction. If two securities are uncorrelated, the movement in one security does not imply a linear movement up or down in the other security.

Estimation risk. Sometimes also called "parameter uncertainty" is the error introduced in portfolio construction process that arises from differences in values in forecasted and realized expected returns, standard deviations, and correlations.

Expected return. The mean of a probability distribution of returns.

Mean-variance optimization. A method to select portfolio weights that provides optimal trade-off between the mean and the variance of the portfolio return for a desired level of risk.

Standard deviation. A statistical measure of dispersion of the observed return, which depicts how widely a stock or portfolio's returns varied over a certain period of time. When a stock or portfolio has a high standard deviation, the predicted range of performance is wide, implying greater volatility.

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